

Expectations

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Exercise 1. Compute the expectation of a random normal $\mathcal{N}(\mu, \sigma^2)$.

Exercise 2. Compute the expectation and the variance of a uniform distribution in the interval $[0, 2]$.

Exercise 3. Let X be a random variable with mean μ and variance σ^2 . Compute

1. $E(2X - 3)$ and $V(2X - 3)$.
2. $V(5 - X)$.
3. $E((X - 2)(X + 1))$.

Exercise 4. If X is a Poisson random variable of parameter λ , show that

1. $E(X) = \lambda$.
 2. $E(X(X - 1)) = \lambda^2$.
- ! Compute $E(X^2)$ and $E((X - E(X))^2)$

Exercise 5. Compute the expectation of a continuous random variable X with distribution:

1. $f(x) = 6x(1 - x)$ in $[0, 1]$ -
2. $f(x) = \frac{3}{x^4}$ if $x > 1$.
3. $f(x)$ if it's proportional to x^2 whenever $0 < x < 1$ and zero otherwise.

Exercise 6. Compute the expectation of a discrete random variables such that $P(X = 1) = \frac{1}{4}$, $P(X = 2) = \frac{1}{2}$ and $P(X = 1000) = \frac{1}{4}$.

Exercise 7. Compute the expectation and the variance of a random variable with distribution $f(x) = \lambda^2 x e^{-\lambda x}$ for $x > 0$.

Exercise 8. ! Let X be a random variable with uniform distribution in $[0, 1]$. Compute $E(e^{5X})$. Can you find $E(1/X)$?

Exercise 9. Let X and Y be two independent random variables such that $E(X) = 2$, $E(X^2) = 6$, $E(Y^2) = 13$ and $E(Y = 3)$. Compute

- $E(X^2 - 3X + 2)$, $E((X + 1)^2)$, $E((X - E(X))^2)$ and $E(X^2) - E(X)^2$.
- $E(X + Y)$, $E(2XY)$, $E((3X - Y)^2)$, $E(3X - Y)^2$ and $E(X | Y = 2 =)$.